IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41})+P_{12}$$
$$=1-(G_2H_1+G_3H_2+G_4H_3+G_6H_2H_3)+G_2G_4H_1H_3$$
$$=1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3$$

Since there is no part of graph which is non-touching with forward path-1 and 2,

$$\Delta_1=\Delta_2=1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T=\frac{1}{\Delta}\sum_K P_K\Delta_K=\frac{1}{\Delta}(P_1\Delta_1+P_2\Delta_2)\ \text{(Number of forward paths is two and so K = 2)}$$

$$=\frac{G_1G_2G_3G_4G_5+G_1G_2G_5G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3}$$

EXAMPLE 1.30

Convert the given block diagram to signal flow graph and determine C(s)/R(s).

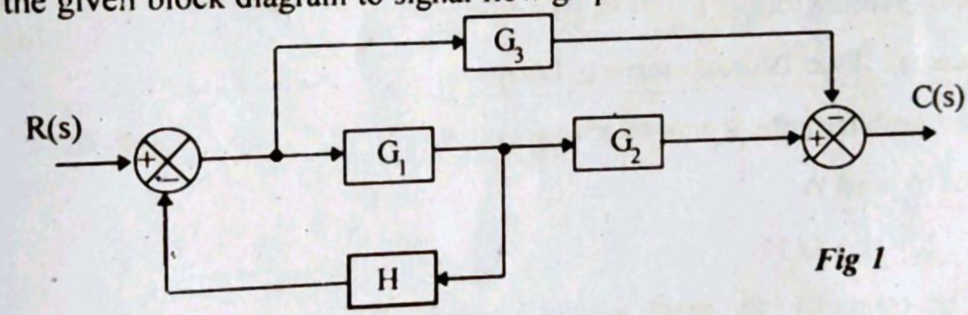

Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

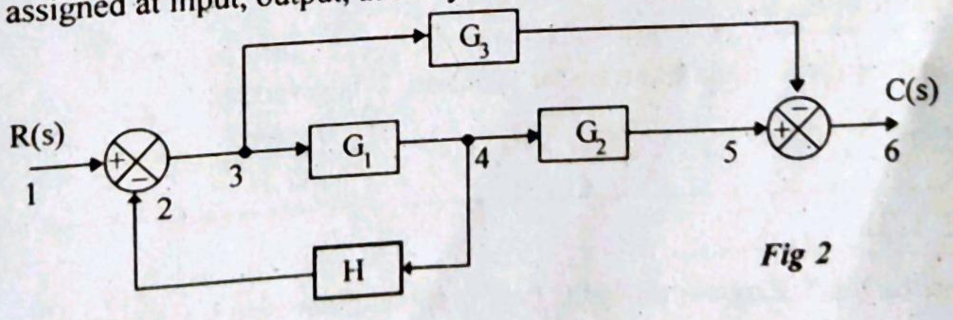

Fig 2

The signal flow graph of the above system is shown in fig 3.

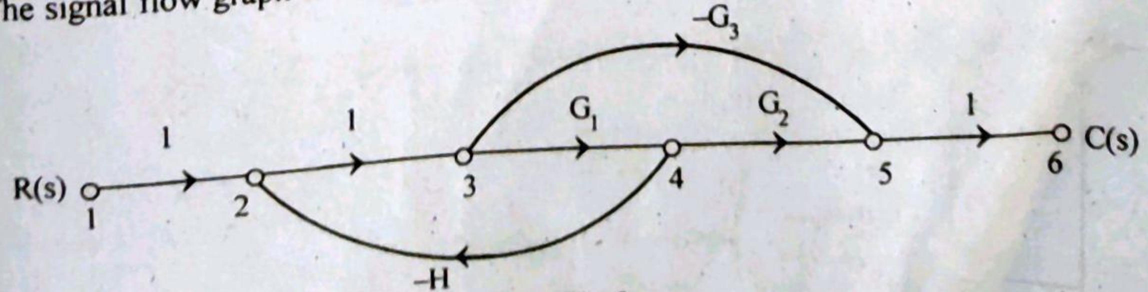

Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore K=2$

Let the forward path gains be P_1 and P_2.

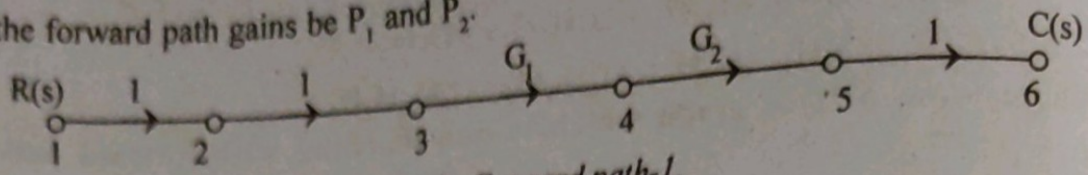

Fig 4 : Forward path-1.

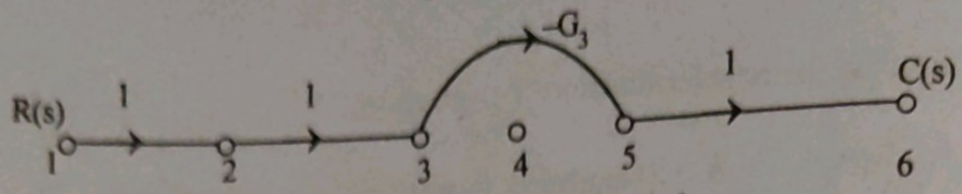

Fig 5 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2$

Gain of forward path-2, $P_2 = -G_3$

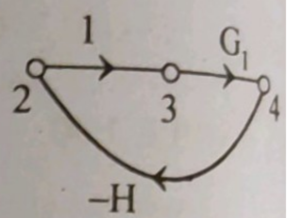

Fig 6 : loop-1.

II. Individual Loop Gain

There is only one individual loop. Let the individual loop gain be P_{11}.

Loop gain of individual loop-I, $P_{11} = -G_1H$.

III. Gain Products of Two Non-touching Loops

There are no combinations of non-touching Loops.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}] = 1 + G_1H$$

Since there are no part of the graph which is non-touching with forward path-1 and 2,

$$\Delta_1 = \Delta_2 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{G_1G_2 - G_3}{1+G_1H}$$

EXAMPLE 1.31

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

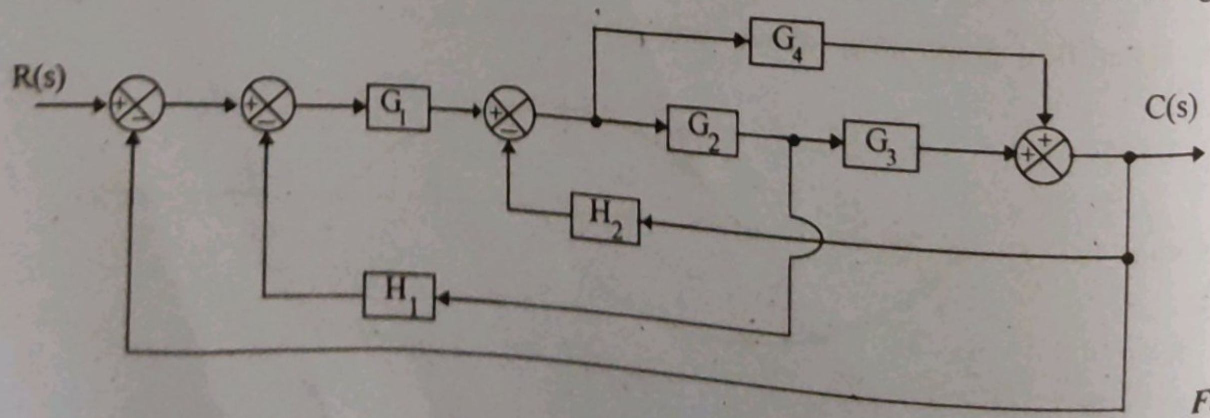

Fig 1

SOLUTION

The nodes are assigned at input, ouput, at every summing point & branch point as shown in fig 2.

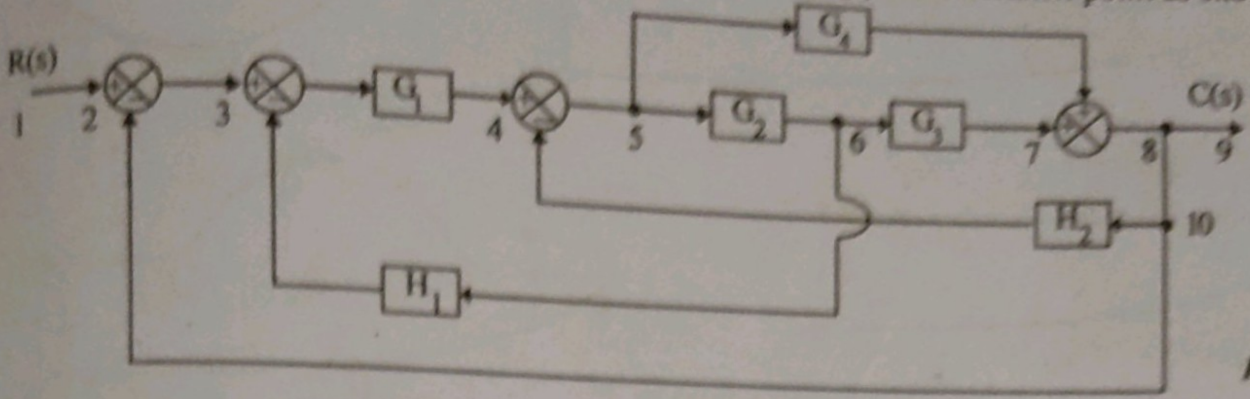

Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

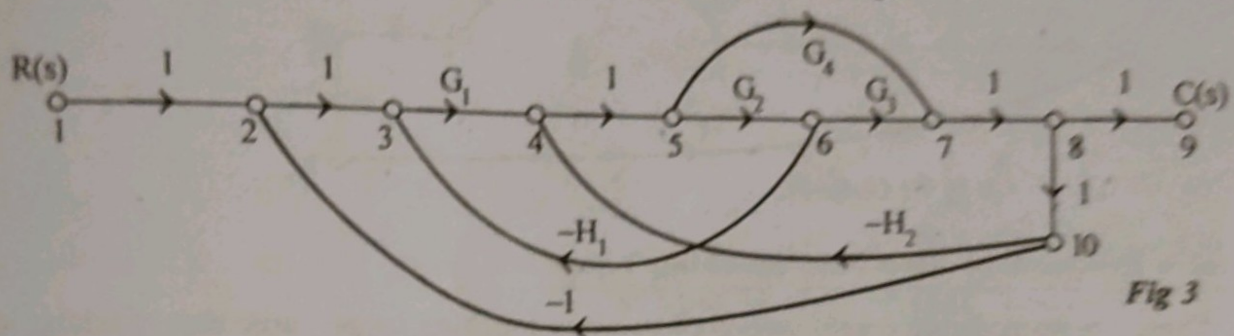

Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore K=2$.

Let the gain of the forward paths be P_1 and P_2.

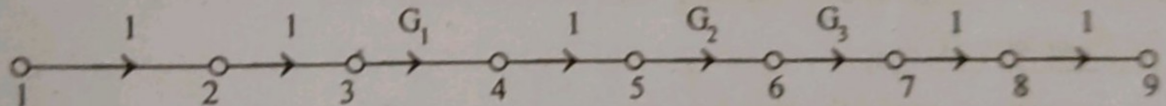

Fig 4 : Forward path-1.

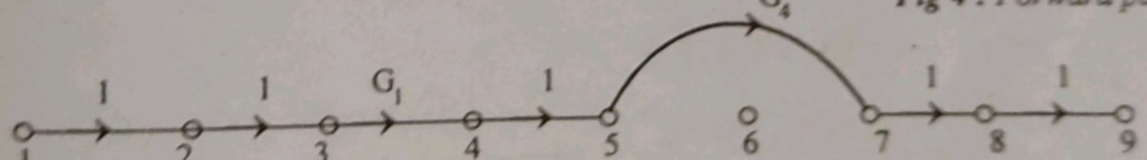

Fig 5 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_1G_4$

II. Individual Loop Gain

There are five individual loops. Let the individual loop gain be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

Loop gain of individual loop-1, $P_{11} = -G_1G_2G_3$

Loop gain of individual loop-2, $P_{21} = -G_2G_1H_1$

Loop gain of individual loop-3, $P_{31} = -G_2G_3H_2$

Loop gain of individual loop-4, $P_{41} = -G_1G_4$

Loop gain of individual loop-5, $P_{51} = -G_4H_2$

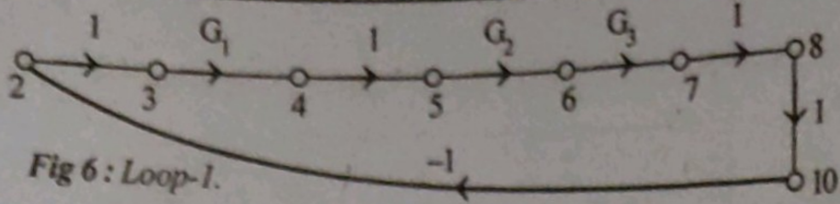

Fig 6 : Loop-1.

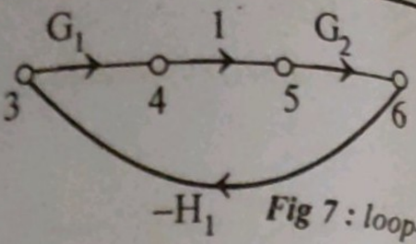

Fig 7 : loop-2.

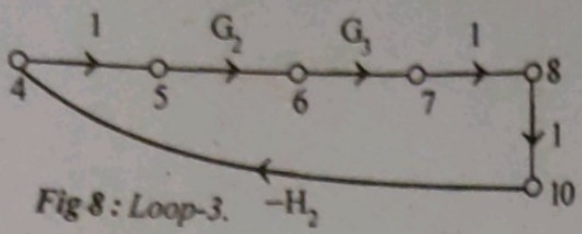

Fig 8 : Loop-3.

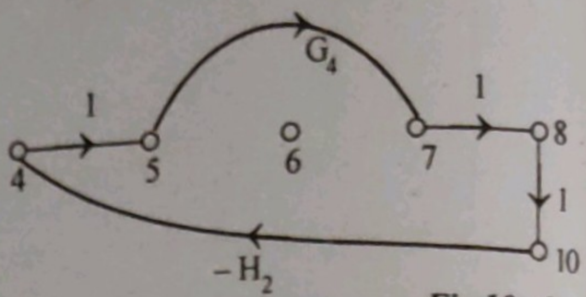

Fig 10 : Loop-5.

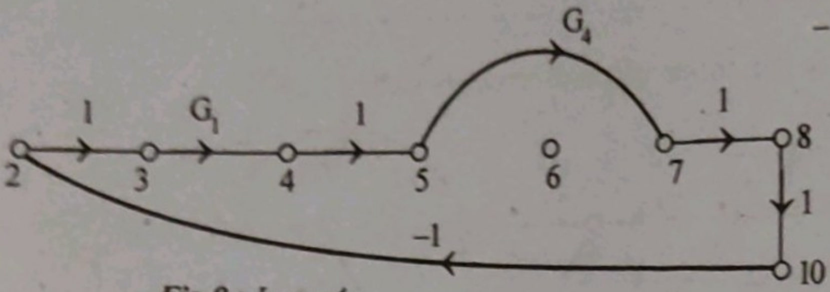

Fig 9 : Loop-4.

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_k

$$\Delta = 1-[P_{11}+P_{21}+P_{31}+P_{41}+P_{51}] = 1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_1G_4+G_4H_2$$

Since no part of graph is non touching with forward paths-1 and 2, $\Delta_1 = \Delta_2 = 1$.

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_1G_4+G_4H_2}$$

EXAMPLE 1.32

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

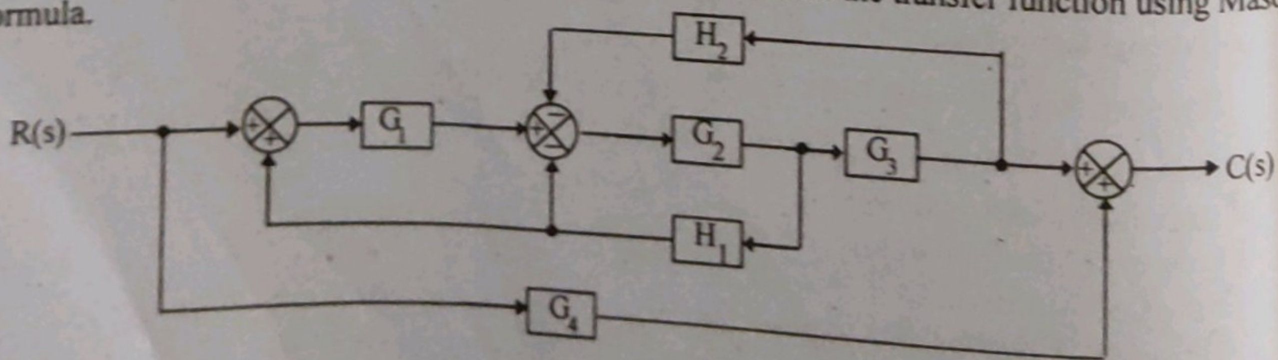

Fig 1

LUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

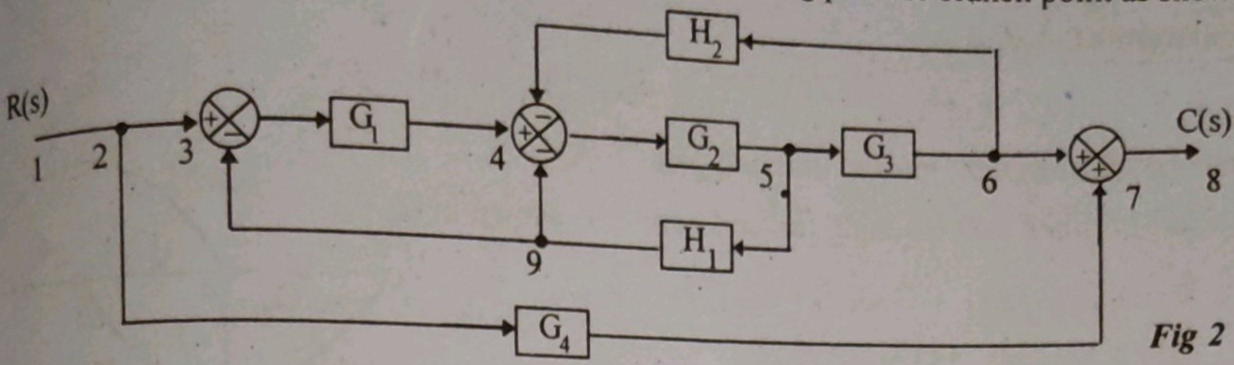

Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

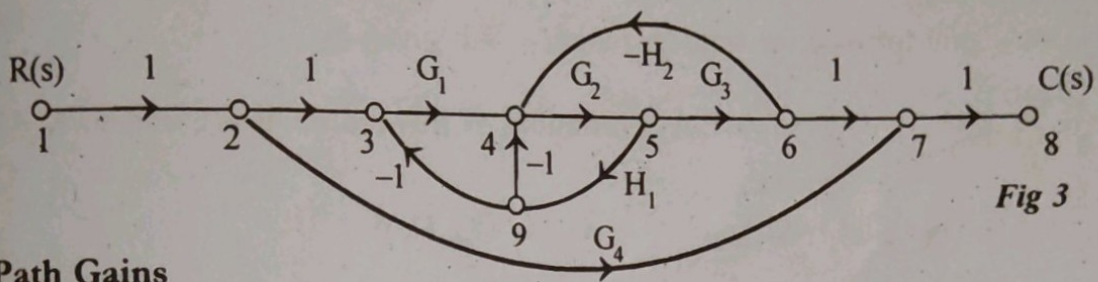

Fig 3

I. Forward Path Gains

There are two forward path, $\therefore$ K=2.

Let the forward path gains be P_1 and P_2.

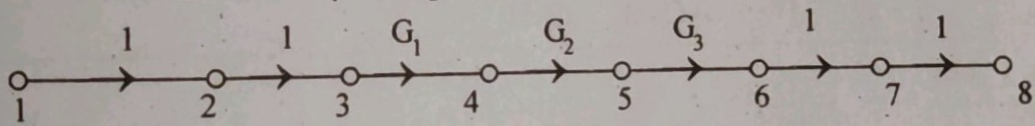

Fig 4 : Forward path-1.

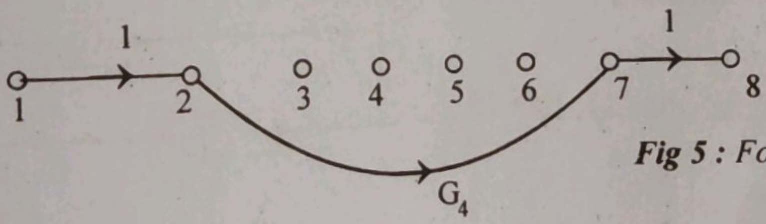

Fig 5 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_4$

II. Individual Loop Gain

There are three individual loops with gains P_{11}, P_{21} and P_{31}.

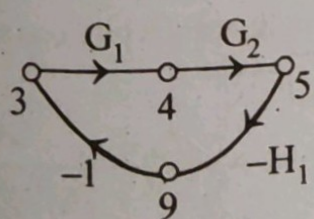

Fig 6 : loop-1.

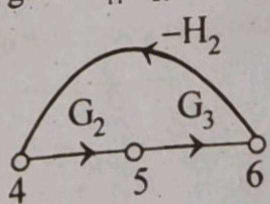

Fig 7 : loop-2.

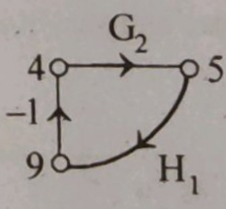

Fig 8 : loop-3.

Gain of individual loop-1, $P_{11} = -G_1G_2H_1$

Gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Gain of individual loop-3, $P_{31} = -G_2H_1$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two-non touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11} + P_{21} + P_{31}] = 1 + G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

Since no part of graph touches forward path–1, $\Delta_1 = 1$.

The part of graph non touching forward path–2 is shown in fig 9.

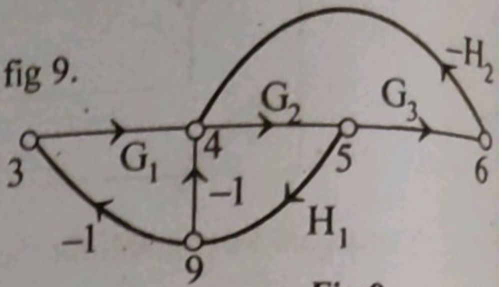

Fig 9

$$\therefore \Delta_2 = 1 - [-G_1G_2H_1 - G_2G_3H_2 - G_2H_1]$$
$$= 1 + G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] \text{ (Number of forward paths is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4(1 + G_1G_2H_1 + G_2G_3H_2 + G_2H_1)]$$

$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4 + G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1]$$

$$= \frac{G_1G_2G_3 + G_4 + G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1}{1 + G_1G_2H_1 + G_2G_3H_2 + G_2H_1}$$

EXAMPLE 1.33

Draw a signal flow graph and evaluate the closed loop transfer function of a system whose block diagram is shown in fig 1.

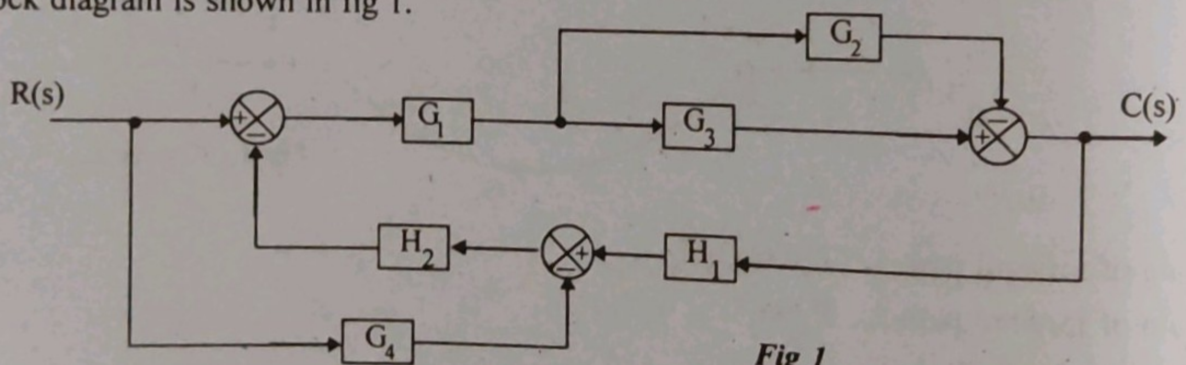

Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

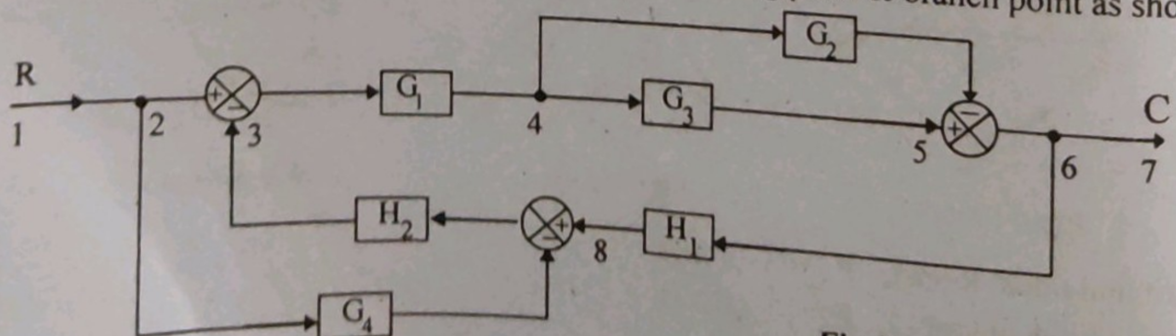

Fig 2

The signal flow graph for the block diagram of fig 2, is shown in fig 3.

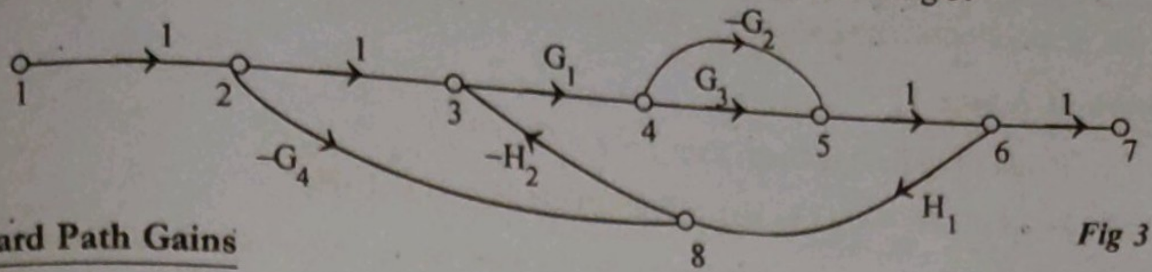

Fig 3

I. Forward Path Gains

There are four forward paths, $\therefore K = 4$

Let the forward path gains be P_1, P_2, P_3 and P_4.

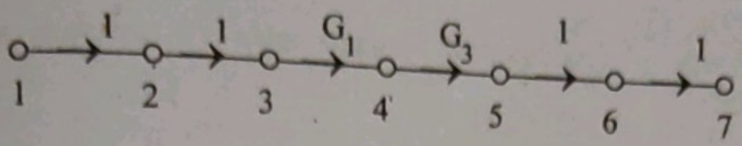

Fig 4 : Forward path-1.

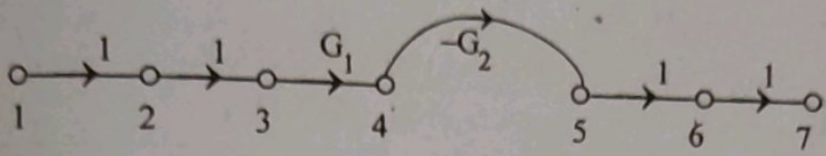

Fig 5 : Forward path-2.

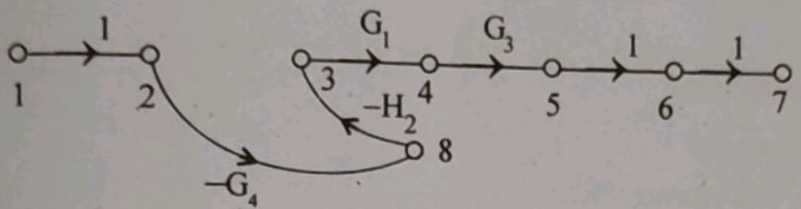

Fig 6 : Forward path-3.

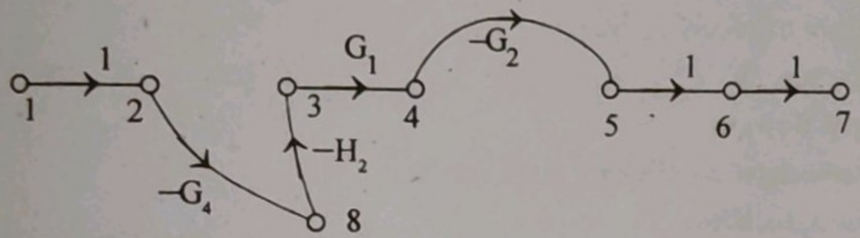

Fig 7 : Forward path-4.

Gain of forward path-1, $P_1 = G_1G_3$

Gain of forward path-2, $P_2 = -G_1G_2$

Gain of forward path-3, $P_3 = G_1G_3G_4H_2$

Gain of forward path-4, $P_4 = -G_1G_2G_4H_2$

II. Individual Loop Gain

There are two individual loops, let individual loop gains be P_{11} and P_{21}.

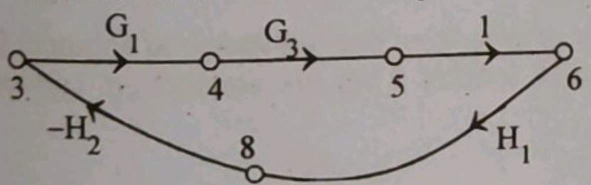

Fig 8 Loop-1.

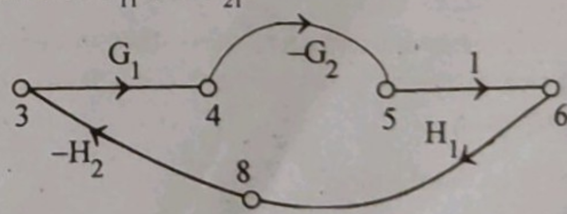

Fig 9 : Loop-2.

Loop gain of individual loop-1, $P_{11} = -G_1G_3H_1H_2$

Loop gain of individual loop-2, $P_{21} = G_1G_2H_1H_2$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [\text{ sum of individual loop gain }] = 1 - (P_{11} + P_{21})$$
$$= 1 - [-G_1G_3H_1H_2 + G_1G_2H_1H_2] = 1 + G_1G_3H_1H_2 - G_1G_2H_1H_2$$

Since no part of graph is non touching with the forward paths, $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$.

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{P_1 + P_2 + P_3 + P_4}{\Delta} \text{ (Number of forward paths is 4 and so K = 4)}$$
$$= \frac{G_1G_3 - G_1G_2 + G_1G_3G_4H_2 - G_1G_2G_4H_2}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}$$
$$= \frac{G_1(G_3 - G_2) + G_1G_4H_2(G_3 - G_2)}{1 + G_1H_1H_2(G_3 - G_2)} = \frac{G_1(G_3 - G_2)(1 + G_4H_2)}{1 + G_1H_1H_2(G_3 - G_2)}$$

1.13 THERMAL SYSTEM

List of symbols used in thermal system.

q	=	Heat flow rate, Kcal/sec.
θ_1	=	Absolute temperature of emitter, °K.
θ_2	=	Absolute temperature of receiver, °K.
Δθ	=	Temperature difference, °C.
A	=	Area normal to heat flow, m^2.
K	=	Conduction or Convection coefficient, Kcal/sec-°C.
K_r	=	Radiation coefficient, Kcal/sec-°C.
H	=	K/A=Convection coefficient, Kcal/m^2-sec-°C.
K	=	Thermal conductivity , Kcal/m-sec-°C.
ΔX	=	Thickness of conductor, m.
R	=	Thermal resistance, °C-sec/Kcal.
C	=	Thermal capacitance, Kcal/°C.

HEAT FLOW RATE

Thermal systems are those that involve the transfer of heat from one substance to another. There are three different ways of heat flow from one substance to another. They are conduction, convection and radiation.

For conduction,

$$\text{Heat flow rate, } q = K\,\Delta\theta = \frac{KA}{\Delta X} \quad .....(1.36)$$

For convection,

$$\text{Heat flow rate, } q = K\,\Delta\theta = HA\,\Delta\theta \quad .....(1.37)$$